LETTERS TO THE EDITOR

SOLUTIONS OF BOUNDARY-LAYER EQUATIONS FOR A NEWTONIAN FLUID ON A PLATE

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The equation for the boundary layer on a plate for a fluid subject to the exponential rheological law derived in [1] has the form

$$n(1+n)f''' + f(f'')^{2-n} = 0, \quad (1)$$

where f is defined by

$$u = f'(\eta), \quad \eta = yx^{-\frac{1}{1+n}}, \quad (2)$$

where x and y are dimensionless coordinates along and across the plate; u is the dimensionless longitudinal velocity; n is the exponent in the rheological law; the prime denotes the derivative with respect to η.

The boundary conditions are given in conventional form:

$$f = f' = 0 \quad \text{when} \quad \eta = 0, \quad (3)$$

$$f' = 1 \quad \text{when} \quad \eta = \infty. \quad (4)$$

Further, in [1] it is maintained that the function satisfying Eq. (1) and Eqs. (3) and (4) can be found only when $n < 2$. For $n \geq 2$ it is proposed that condition (4) be replaced by

$$f' = 1 \text{ and } f'' = 0 \quad \text{when} \quad \eta \geqslant \eta_1, \quad (5)$$

where η_1 is the finite thickness of the boundary layer. This contention is cited in [2].

We will demonstrate that condition (5) must be used instead of (4) even when $n > 1$.

Let us write Eq. (1) in the form

$$n(1+n)f'''(f'')^{n-2} \equiv \equiv \frac{n(1+n)}{n-1}\frac{d}{d\eta}(f'')^{n-1} = -f. \quad (6)$$

Integrating (6) twice, with consideration of (3), we obtain

$$f' = C\int_0^\eta d\xi \exp\left(-\frac{1}{2}\int_0^\xi f(\nu)\,d\nu\right) \quad (n = 1), \quad (7)$$

$$f' = \int_0^\eta d\xi\left[C_1 - \frac{n-1}{n(1+n)}\int_0^\xi f(\nu)\,d\nu\right]^{\frac{1}{n-1}} \quad (n \neq 1). \quad (8)$$

The constants C and C_1 must be determined from (4) or (5).

Since $f(\infty) = \infty$, it is easy to see that, for $n \leq 1$, the integrals of (7) and (8) as $\eta \to \infty$ are limited, and C and C_1 are determined from (4). When $n > 1$ as $\eta \to \infty$, the integral (8) diverges and C_1 can be determined only from condition (5).

Thus, when $n \leq 1$ the boundary layer described by (1) is asymptotic in nature, while when $n > 1$ the boundary layer has finite thickness. The same result is derived by an approximate method in [3].

REFERENCES

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